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DEPARTMENT OF EDUCATION (S)
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# CLASS – X
# HIGHER MATHEMATICS
# CHAPTER – 1
# BINARY OPERATIONS

- **Binary operation**: Let $A$ be any non- empty set. Then, a function $*: A \times A \to A$ is called a binary operation or binary composition or an internal composition on the set $A$.
- **External binary operation**: Let $A$ and S be any non- empty sets. Then, a function is$*: A \times S \to S$ called an external binary operation on S over $A$.
- **Algebraic Structure**

A set equipped with one or more binary operations (internal or external) is known as an algebraic structure.

Note: Let * be any operation on a non-empty set $A$, then * is called binary operation on $A$, if $a*b \in A$, for every $a,b \in A$. It is called **closure law** and we say that ' $A$ is closed with respect to * or " $A$ satisfies closure law with respect to * '.

- **Associativity and Commutativity**

Let * be a binary operation on a non-empty set $A$, then

(i) * is called **commutative** , if $a*b = b*a$, for every $a,b \in A$.

(ii) * is called **associative**, if $(a*b)*c = a*(b*c)$, for every $a,b,c \in A$.

- **Identity Element**

For an algebraic structure (A, $*$) ,an element $e \in A$ (if it exists) is called an **identity element** , if $a*e = e*a = a$, for every $a \in A$.

- **Inverse of an Element**

Let * be a binary operation on a non-empty set $A$ and let $e \in A$ be the identity element of * on $A$. Then, an element $a \in A$ is called **invertible (or inversible)**, if there exists an element $b \in A$ such that $a*b = b*a = e$. The element $b$ is called the inverse of $a$ and it denoted as $a^{-1}$.

- **Distributivity**

Let * and $\circ$ be two binary operations on a set $A$. Then , * is distributive over $\circ$, if $a*(b \circ c) = (a*b) \circ (a*c)$ (left distributive law)

and $(b \circ c)*a = (b*a) \circ (c*a)$ ,(right distributive law) for every $a,b,c \in A$.

- **Theorem**: The identity element for an algebraic structure, if it exists, is unique.

- **Theorem**: If ( $A$ , *) is an algebraic structure with identity, in which the binary operation * is associative, then the inverse of an element of $A$, if it exists, is unique.
- **Theorem**: Let ( $A$ , *) is an algebraic structure with identity, in which the binary operation * is associative. If $a$ and $b$ are two invertible elements of $A$, then $a*b$ is also invertible and $(a*b)^{-1} = b^{-1}*a^{-1}$.
- **Subsets closed under a binary operation**

Let * be a binary operation on a set $A$ and $H$ be a subset of $A$. Then , $H$ is said to be closed under *, if for every pair $(a,b) \in H \times H \Rightarrow a*b \in H$.

- **Composition Table (or Operation Table)**

Let $A$ be a non-empty set and let * be an operation on $A$. Then , we can completely describe the operation with the help of a table called ***composition table*** (or ***operation table***). Let $A = \{a_1, a_2, ..., a_n\}$ be a finite set and * be an operation on $A$ .Then we can construct composition table as given below:

| * | $a_1$ | $a_2$ | . . . | $a_i$ | . . . | $a_n$ |
|---|---|---|---|---|---|---|
| $a_1$ | $a_1*a_1$ | $a_1*a_2$ | . . . | $a_1*a_i$ | . . . | $a_1*a_n$ |
| $a_2$ | $a_2*a_1$ | $a_2*a_2$ | . . . | $a_2*a_i$ | . . . | $a_2*a_n$ |
| .<br>.<br>. | .<br>.<br>. | .<br>.<br>. | . . . | .<br>.<br>. | . . . | .<br>.<br>. |
| $a_i$ | $a_i*a_1$ | $a_i*a_2$ | . . . | $a_i*a_i$ | . . . | $a_i*a_n$ |
| .<br>.<br>. | .<br>.<br>. | .<br>.<br>. | . . . | .<br>.<br>. | . . . | .<br>.<br>. |
| $a_n$ | $a_n*a_1$ | $a_n*a_2$ | . . . | $a_n*a_i$ | . . . | $a_n*a_n$ |

- The elements of the left-most column are called heads of the corresponding rows.
- The elements of the top-most row are called heads of the corresponding columns.
  We can infer the following result from the composition table:
  (i) * is a binary operation if all the entries of the table belong to the set $A$.
  (ii) * is commutatve if the composition table is symmetric about the diagonal joining the upper left corner and the lower right corner.
  (iii) An element $e \in A$ is the identity element if the row headed by $e$ coincides with the top-most row and the column headed by $e$ coincides with the left-most column.
  (iv) Let $e \in A$ be the identity element. If $e$ appears in the entry of the table (except the top-most row and the left-most column), then the heads of that particular row and column are **invertible** and are inverses of each other.

- **Some Special Operations**

### 1. Addition modulo $n$

Let $n$ be a positive integer, we define the operation 'addition modulo $n$' denoted by '$+_n$' as follows:

$a +_n b$ =the least non-negative remainder when $a + b$ is divided by $n$; $a, b \in Z$

For example,

$2 +_6 5$ = the least non-negative remainder when $2 + 5$ $i.e.$ 7 is divided by 6

$= 1$

### 2. Multiplication modulo $n$

Let $n$ be a positive integer, we define the operation ' multiplication modulo $n$' denoted by '$\times_n$' as follows:

$a \times_n b$ = the least non-negative remainder when $a \times b$ is divided by $n$; $a, b \in Z$.

For example,

$2 \times_6 5$ = the least non-negative remainder when $2 \times 5$ $i.e.$ 10 is divided by 6

$= 4$

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