CHAPTER 2
SEQUENCES, A.P., G.P. and H.P.

SEQUENCE : A sequence is an arrangement of numbers in a definite order according to some rules.

E.g. :- 2, 4, 6, 8, ……….. is a sequence.

A sequence is said to be finite if the number of its elements is finite, otherwise it is said to be infinite.

A finite sequence $a_1, a_2, a_3, ..., a_k$ is denoted by $\{a_n\}^k_{n=1}$ and an infinite sequence $a_1, a_2, a_3........, a_{n,.....}$ is denoted by $\{a_n\}^{\alpha}_{n=1}$ or simply by $\{a_n\}$, where a_n is the n^{th} term of the sequence.

Arithmetic Progression (A.P.): A sequence $\{a_n\}$ is called an arithmetic progression (AP) if there exists a no. d such that $a_{n+1} - a_n = d\ \forall n \in N$. The number d is called the common difference (c.d.) of the AP.

Notes:

1) The general term or n^{th} term (a_n) of an A.P. whose first term is a andcommon difference is d,is given by $a_n = a + (n-1)d$

2) Sum of the first n terms (s_n) of an A.P. is given by

$$S_n = \frac{n}{2}[a+l], \text{ or } S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

Arithmetic Mean (AM) : The arithmetic mean (AM) between two numbers a and b is given by

$$A.M. = \frac{1}{2}(a+b)$$

Geometric Progression (GP): The sequence $\{a_n\}$ is called a geometric progression (GP) if there exists a non-zero number r such that $\frac{a_{n+1}}{a_n} = r,\ \forall n \in N$.

The number r is called the common ratio (c.r.) of the GP.

Notes: 1)The general term or n^{th} term a_n of a G.P. whose first term is a and common ratio is r, is given by

$$a_n = ar^{n-1}$$

2) The sum of the first n terms, S_n of a G.P. is given by

(i) $S_n = \frac{a(r^n-1)}{r-1}$ if $r > 1$

(ii) $S_n = \frac{a(1-r^n)}{1-r}$ if $r < 1$

and (iii) $S_n = na$, if $r = 1$

.Geometric Mean (G.M.): If a, x, b are in GP, then x is the geometric mean between a and$b$.

$\therefore$ GM between a and b is given by, $x = \sqrt{ab}$

Harmonic Progression (HP): A sequence $\{a_n\}$ is called a harmonic progression if the sequence $\left\{\frac{1}{a_n}\right\}$ is an AP. i.e. if there exists a number d such that $\frac{1}{a_{n+1}} - \frac{1}{a_n} = d, \forall n \in N$.

Harmonic Mean (HM) : If H be the harmonic mean between a and$b$, then a, H, b are in HP and consequently $\frac{1}{a}, \frac{1}{H}, \frac{1}{b}$ are in AP.

HM between a and $b = \frac{2ab}{a+b}$

Relation between AM, G.M. and HM:

$i) A.M., G.M.$ and $H.M.$ are in $G.P.$

$ii) AM > GM > HM$ (for two unequal quantities)

SERIES

Sum of some important finite series are :

(i) $1+2+3+........+n = \frac{n(n+1)}{2}$

(ii) $1^2+2^2+3^2+..........+n^2 = \frac{n(n+1)(2n+1)}{6}$

(iii) $1^3+2^3+3^3+......+n^3 = \left[\frac{n(n+1)}{2}\right]^2$

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