CHAPTER 4
BINOMIAL THEOREM

NOTES

BINOMIAL EXPRESSIONS:

Algebraic expressions having two terms are called binomial expressions.

E.g.: $(x+3),(3x-2y)$ etc.

BINOMIAL THEOREM:

If a and x be any two real numbers and n be any positive integer, then

$$(a+x)^n = {}^nC_0a^n + {}^nC_1a^{n-1}x + {}^nC_2a^{n-2}x^2 + \dots\dots + {}^nC_ra^{n-r}x^r + \dots + {}^nC_nx^n$$

DEDUCTIONS:

In the binomial expansion,

$$(a+x)^n = {}^nC_0a^n + {}^nC_1a^{n-1}x + {}^nC_2a^{n-2}x^2 + \dots\dots + {}^nC_ra^{n-r}x^r + \dots + {}^nC_nx^n$$

(i) If x is replaced by $-x$

$$(a-x)^n = {}^nC_0a^n - {}^nC_1a^{n-1}x + {}^nC_2a^{n-2}x^2 - \dots\dots + (-1)^r\,{}^nC_ra^{n-r}x^r + \dots + (-1)^n\,{}^nC_nx^n$$

(ii) If $a=1$, we get

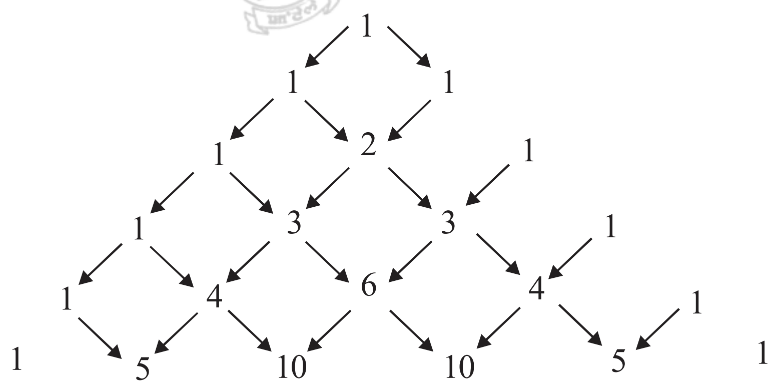
$$(1+x)^n = 1 + {}^nC_1x + {}^nC_2x^2 + \dots\dots + {}^nC_rx^r + \dots + x^n$$

(iii) If $a=1$ and x is replaced by $-x$, we get

$$(1-x)^n = 1 - {}^nC_1x + {}^nC_2x^2 - \dots\dots + (-1)^r\,{}^nC_rx^r + \dots + (-1)^n\,x^n$$

PASCAL'S TRIANGLE :

The binomial co-efficients can be arranged in the form of triangle as follows:



General term: In the expansion $(a+x)^n$, the general term is denoted by T_{r+1} and is given by

$T_{r+1} = {}^nC_r a^{n-r} x^r$

Middle term of a binomial expansion:

1. When n is even

When n is even, the number of terms in the expansion of $(a+x)^n$ is $(n+1)$ which is odd.

$\therefore$ The middle term is $\left(\frac{n}{2}+1\right)^{th}$ term. Hence $T_{\frac{n}{2}+1}$ is the middle term.

2. When n is odd.

The number of terms $(n+1)$ being even, there are two middle terms which are the $\left(\frac{n+1}{2}\right)^{th}$ term and $\left(\frac{n+1}{2}+1\right)^{th}$ term.

Thus, $T_{\frac{n+1}{2}}$ and $T_{\frac{n+3}{2}}$ are two middle terms.

Properties of binomial co-efficients:

i) The sum of all the binomial co-efficientsis 2^n.

i.e. ${}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots\ldots + {}^nC_n = 2^n$

ii) The sum of the binomial co-efficients of odd terms is equal to that of even terms, each being equal to 2^{n-1}.

i.e. ${}^nC_0 + {}^nC_2 + {}^nC_4 + \ldots\ldots = {}^nC_1 + {}^nC_3 + {}^nC_5 + \ldots\ldots = 2^{n-1}$

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