CHAPTER 7
TRIGONOMETRY

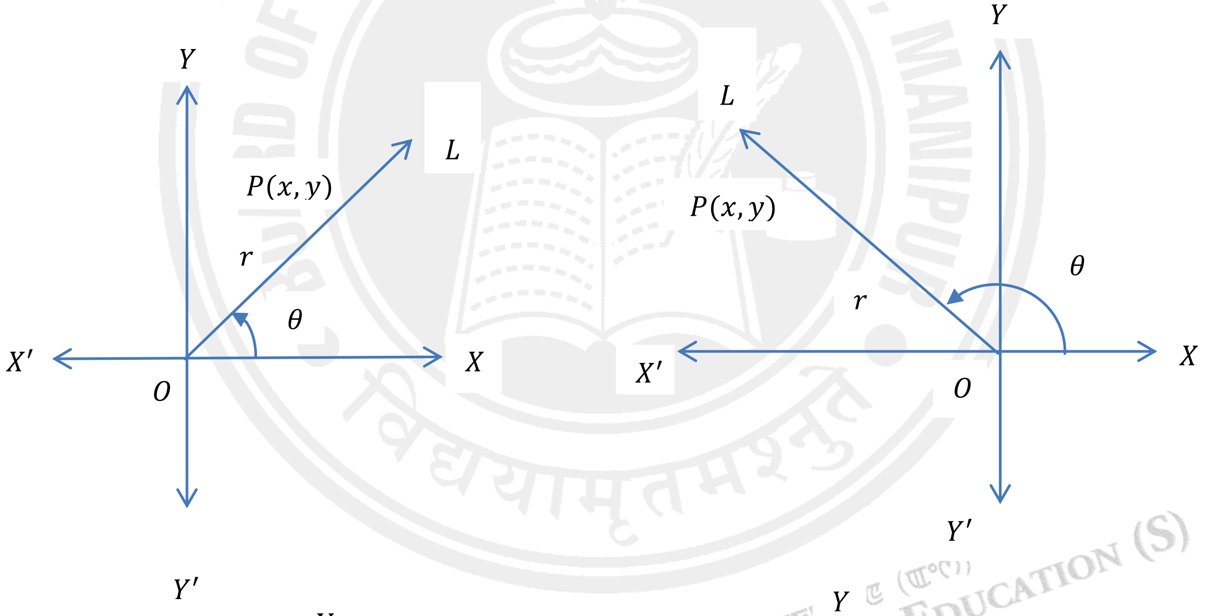
NOTES

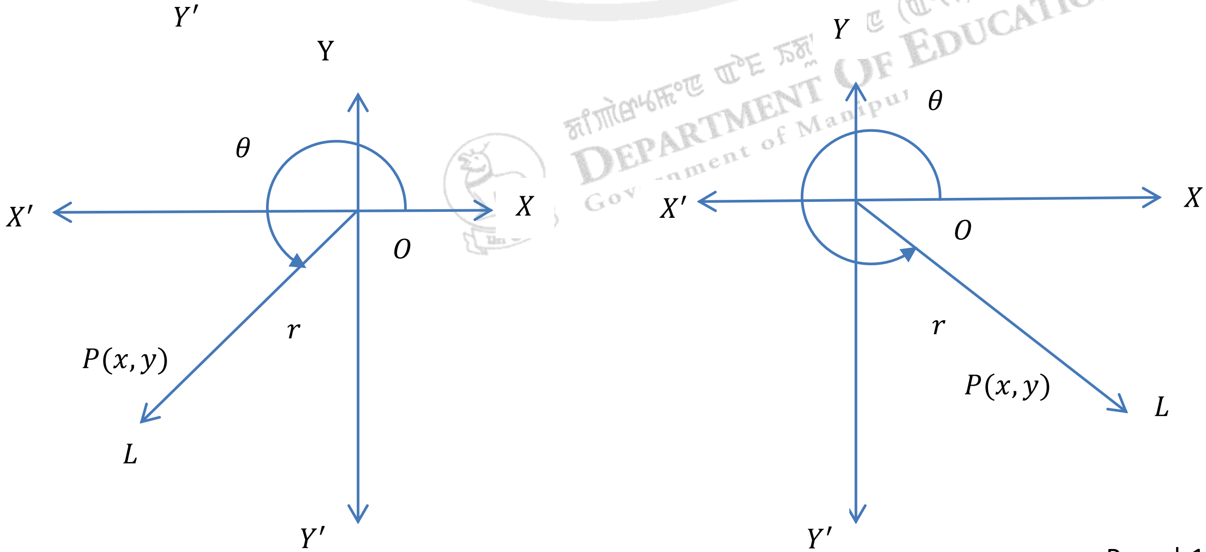
Angle: An angle is a measure of rotation of a given ray about its initial point. The original position of the ray is called the initial side and the final position of the ray after rotation is called the terminal side of the angle. If the direction of rotation is anticlockwise then the angle is taken as positive and if the direction of rotation is clockwise, then the angle is taken as negative.

The measure of an angle is the amount of rotation performed to get the terminal side from the initial side.

Trigonometric Ratios of angles of any sign and magnitude:

Let $P(x, y)$ be any point other than the origin on the final position of a revolving line OL which revolves an angle θ from the initial position OX in a cartesian plane and let $OP = r(> 0)$, then we define trigonometric ratios of θ as





$\sin\theta = \dfrac{y}{r}$; $\quad cosec\theta = \dfrac{r}{y}, (y \neq 0)$

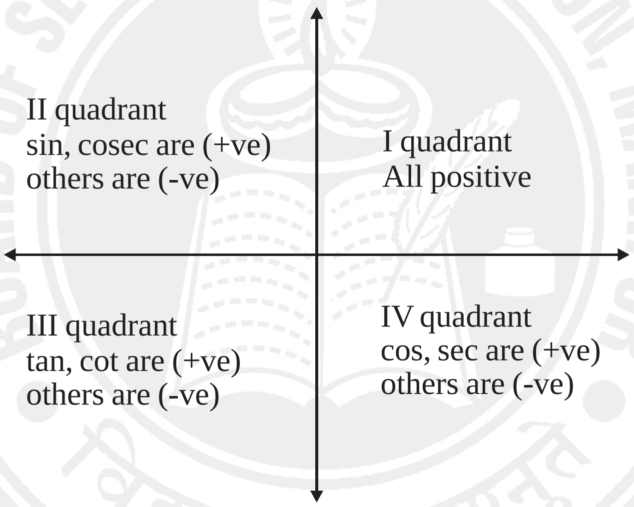
$\cos\theta = \dfrac{x}{r}$; $\quad sec\theta = \dfrac{r}{x}, (x \neq 0)$

$\tan\theta = \dfrac{y}{x}, (x \neq 0)$; $\quad \cot\theta = \dfrac{x}{y}, (y \neq 0)$

Note: These general definitions of trigonometric ratios of θ agree with the definitions given in general Maths when θ is acute.

Signs of the trigonometric ratios in different quadrants:

Quadrant Rule: "all, sin, tan, cos"



Trigonometric ratios of allied angles:

ALLIED (or ASSOCIATED) ANGLES: Two angles are said to be allied when their sum or difference is a multiples of 90^0.

Notes:

1. The angles $-\theta, 90° \pm \theta, 180° \pm \theta, 270° \pm \theta, 360° \pm \theta$, etc. are angles allied to θ (measure in degrees).
2. In general, any angle of the form $n \times 90° \pm \theta$, $n \in Z$, is allied to (associated with) θ.
3. If θ is measured in radians, then the angles allied to θ are $-\theta, \dfrac{\pi}{2} \pm \theta, \pi \pm \theta, \dfrac{3\pi}{2} \pm \theta,\ 2\pi \pm \theta$ etc.

T-ratios of ($-\theta$) in terms of those of θ :

$\sin(-\theta) = -\sin\theta$ $\quad$ $\operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta$

$\cos(-\theta) = \cos\theta$ $\quad$ $\sec(-\theta) = \sec\theta$

$\tan(-\theta) = -\tan\theta$ $\quad$ $\cot(-\theta) = -\cot\theta$

Note: If θ is associated with even multiples of 90^0 or $\frac{\pi}{2}$ then there is no change in the trigonometric ratios. If θ is associated with odd multiples of 90^0 or $\frac{\pi}{2}$ then the trigonometric ratios will change.

T-ratios of $(90^0 - \theta)$ in terms of those of θ :

$\sin(90^0 - \theta) = \cos\theta$ $\quad$ $\cot(90^0 - \theta) = \tan\theta$

$\cos(90^0 - \theta) = \sin\theta$ $\quad$ $\operatorname{cosec}(90^0 - \theta) = \sec\theta$

$\tan(90^0 - \theta) = \cot\theta$ $\quad$ $\sec(90^0 - \theta) = \operatorname{cosec}\theta$

T-ratios of $(90^0 + \theta)$ in terms of those of θ :

$\sin(90^0 + \theta) = \cos\theta$ (as $90^0 + \theta$ lies in II quadrant).

$\cos(90^0 + \theta) = -\sin\theta$

$\tan(90^0 + \theta) = -\cot\theta$

$\cot(90^0 + \theta) = -\tan\theta$

$\sec(90^0 + \theta) = -\operatorname{cosec}\theta$

$\operatorname{cosec}(90^0 + \theta) = \sec\theta$

T-ratios of $(180^0-\theta)$ in terms of those of θ :

$\sin(180^0-\theta)=\sin\theta$, [as $(180^0-\theta)$ lies in II quadrant and

$$(180^0-\theta)=(2\times 90^0-\theta)]$$

$\cos(180^0-\theta)=-\cos\theta$

$\tan(180^0-\theta)=-\tan\theta$

$\cot(180^0-\theta)=-\cot\theta$

$\sec(180^0-\theta)=-\sec\theta$

$\operatorname{cosec}(180^0-\theta)=\operatorname{cosec}\theta$

T-ratios of $(180^0+\theta)$ in terms of those of θ :

$\sin(180^0+\theta)=-\sin\theta$, [as $(180^0+\theta)$ lies in III quadrant]

$\cos(180^0+\theta)=-\cos\theta$

$\tan(180^0+\theta)=\tan\theta$

$\cot(180^0+\theta)=\cot\theta$

$\sec(180^0+\theta)=-\sec\theta$

$\operatorname{cosec}(180^0+\theta)=-\operatorname{cosec}\theta$

T-ratios of $(270^0-\theta)$ in terms of those of θ :

$\sin(270^0-\theta)=-\cos\theta$, [as $(270^0-\theta)$ lies in III quadrant and

$$(270^0-\theta)=(3\times 90^0-\theta)]$$

$\cos(270^0-\theta)=-\sin\theta$

$\tan(270^0-\theta)=\cot\theta$

$\cot(270^0-\theta)=\tan\theta$

$\sec(270^0-\theta)=-\operatorname{cosec}\theta$

$\operatorname{cosec}(270^0-\theta)=-\sec\theta$

T-ratios of $(270^0+\theta)$ in terms of those of θ :

$\sin(270^0+\theta)=-\cos\theta$,[as $(270^0+\theta)$ lies in III quadrant and

$$(270^0+\theta)=(3\times 90^0+\theta)\,]$$

$\cos(270^0+\theta)=\sin\theta$

$\tan(270^0+\theta)=-\cot\theta$

$\cot(270^0+\theta)=-\tan\theta$

$\sec(270^0+\theta)=cosec\,\theta$

$cosec(270^0+\theta)=-sec\theta$

T-ratios of $(360^0-\theta)$ in terms of those of θ :

$\sin(360^0-\theta)=-\sin\theta$, [as $(360^0-\theta)$ lies in IV quadrant and

$$(360^0-\theta)\quad=(4\times 90^0-\theta)\,]$$

$\cos(360^0-\theta)=\cos\theta$

$\tan(360^0-\theta)=-\tan\theta$

$\cot(360^0-\theta)=-\cot\theta$

$\sec(360^0-\theta)=sec\theta$

$cosec(360^0-\theta)=-cosec\,\theta$

Note: The trigonometric ratios of $(360^0+\theta)$ are the same as those of θ and the trigonometric ratios of $(360^0-\theta)$ are the same as those of $(-\theta)$.

Trigonometric Equation:

An equation involving trigonometric ratios of unknown angles (variable) is known as trigonometric equations.

e.g. $\sin 2\theta+\cos\theta=0$

Solution of trigonometric equation:

A solution of a trigonometric equation is the value of the unknown angle (variable) that satisfies the equation.

Note: A trigonometric equation has infinitely many solutions.

Principal solution: The solution θ of a trigonometric equation between

$0° \leq \theta < 360°$ are known as principal solutions.

General solutions: The expression of all the infinitely many solutions of a trigonometric equation in terms of $n \in Z$ is known as general solution.

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