CHAPTER 8
STATICS

NOTES

Statics is that part of mechanics which deals with bodies at rest when acts on by forces or with the relations between the forces which keep a rigid body (or a system of bodies) at rest.

Some Terms and definitions:

(i) Matter: A matter is anything that occupies space and can be perceived by our senses.
(ii) Force: A force is that which changes or tends to change, the state of rest or of uniform motion of a body.
(iii) Rigid body: A rigid body is one whose size and shape do not alter when acted on by any forces whatsoever, so that the distance between any pair of particles in it remains invariable.
(iv) Particle: A particle is a body of infinitely small dimensions. When we speak of a body as a particle, we mean that we are not concerned with its actual dimensions and that we can represent its position simply by a mathematical point.
(v) Equilibrium: If a system of forces acting on a body keeps it at rest, then the forces are said to be in equilibrium.

The Principle of transmissibility of a force:

The effect of a force acting on a rigid body at any point is unaltered if its point of application is transferred to any other point on its line of action, provided the two points are in the body.

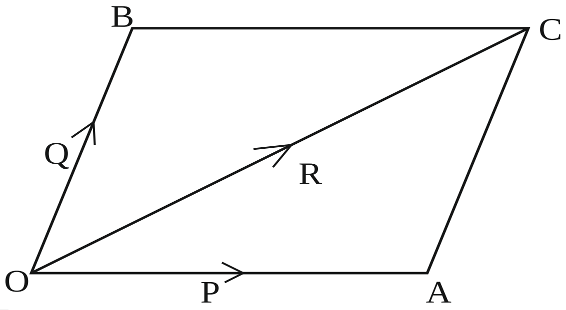
Some Special Forces:

(i) **Weight:** The weight of a body is the force with which the earth attracts the body. It is proportional to the mass of the body and its direction is vertically downwards.
(ii) **Reaction:** According to Newton's third law of motion, "to every action there corresponds an equal and opposite reaction." Thus in a system of two bodies, **A** and **B** if **A** exerts a force **P** (action) on **B**, then the body **B** also exerts an equal force **P** (reaction) in the opposite direction on **A**.
(iii) **Tension:** When a string is used to support a weight or to drag a body, the force exerted is transmitted to the body through the string. Such a force exerted by means of a string is called Tension.

Resultant and Components:

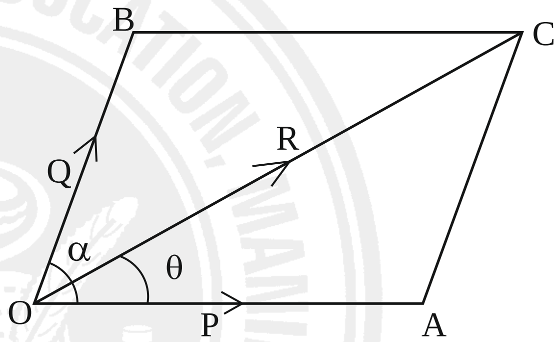
If two or more forces act simultaneously on a rigid body and if a single force can be obtained whose effect on the body is the same as the joint effect of the given forces, then this single force is known as the resultant of the given forces and the given forces, in turn, are called the components of the single resultant force.

Parallelogram of forces:



If two forces acting at a point on a body be represented in magnitude, direction and sense by the two adjacent sides of a parallelogram drawn from an angular point, then their resultant is represented in magnitude, direction and sense by the diagonal of the parallelogram drawn from that point.

Expression for the resultant of two given forces:



Let the two forces P and Q acting at the point O at an angle α to each other be represented by OA and OB respectively. Let $\angle COA = \theta$ which will give the direction of the resultant force, R, represented by OC.

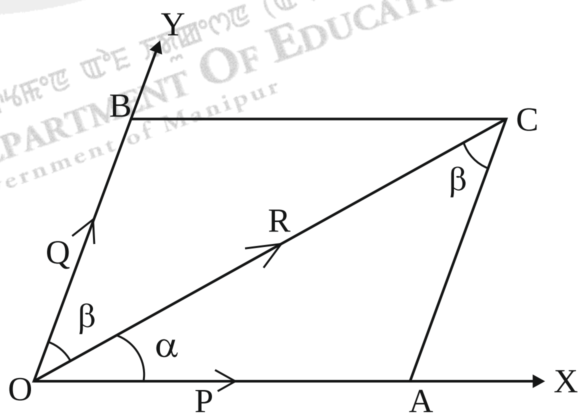
Resultant $R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}$ and

$$\tan\theta = \frac{Q\sin\alpha}{P + Q\cos\alpha}.$$

Corollary: If $\alpha = 90^\circ$ i.e. the two forces P and Q are perpendicular to each other then $R = \sqrt{P^2 + Q^2}$ and $\tan\theta = \frac{Q}{P}$.

The greatest and the least values of the resultant is $R = P + Q$ and $R = |P - Q|$ respectively.

Resolution of a given force into two components:



Let OC represent the given force R and OX and OY be two given directions making angles α and β respectively with OC, on opposite side of it. Then components of R along OX and OY are $\frac{R\sin\beta}{\sin(\alpha+\beta)}$ and $\frac{R\sin\alpha}{\sin(\alpha+\beta)}$ respectively.

Note: If the two components of a given force are along two perpendicular direction, then resolved parts along OX and OY are $P = R\cos\alpha$ and $Q = R\sin\alpha$.

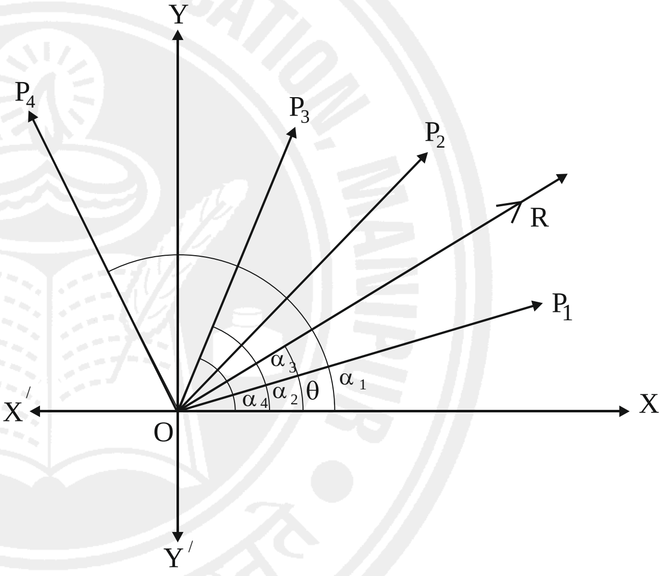
Theorem: The algebraic sum of the resolved parts of any two forces acting at a point, along any direction, is equal to the resolved part of their resultant, in that direction.

Triangle of forces: If three forces acting at a point be such as can be represented in magnitude, direction and sense (but not in position) by the three side of a triangle taken in order, and then the forces are in equilibrium.

Converse of Triangle of forces: If three forces acting at a point be in equilibrium, then they can be represented in magnitude, direction and sense by the three sides of a triangle, taken in order.

Resultant of several coplanar forces simultaneously acting at a point:

Let a number of coplanar forces $P_1, P_2, P_3, P_4, ...etc$ be simultaneously acting at a point O and let their directions makes respectively angles $\alpha_1, \alpha_2, \alpha_3, \alpha_4 ...etc$ with a suitably chosen direction OX in the plane. And let OY be perpendicular to OX. Let R be the resultant of the given forces and let it make an angle θ with OX.



Resolved part of R along OX and OY are equal to the algebraic sum of the resolved parts of the component forces along the same direction, so

$$R\cos\theta = P_1\cos\alpha_1 + P_2\cos\alpha_2 + P_3\cos\alpha_3 + P_4\cos\alpha_4 + \ ... \ = X \text{ (say)}$$

And, $$R\sin\theta = P_1\sin\alpha_1 + P_2\sin\alpha_2 + P_3\sin\alpha_3 + P_4\sin\alpha_4 + \ ... \ = Y \text{ (say)}$$

Hence, $R = \sqrt{X^2 + Y^2}$ ----- (i)

And $\tan\theta = \dfrac{Y}{X}$ ------ (ii)

Equations (i) and (ii) give respectively the magnitude and direction of the resultant.

Corollary: When $X = 0$ and $Y = 0$ then $R = 0$.

Therefore, the forces are in equilibrium if the sum of their resolved parts along two perpendicular directions OX and OY vanish separately.

Conversely, if the forces are in equilibrium i.e. $R=0$ then $X=0$ and $Y=0$.

Thus the necessary and sufficient condition for the equilibrium of the concurrent and coplanar forces are $X=0$ and $Y=0$.

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