CHAPTER 11
COORDINATE GEOMETRY

NOTES

❖ ***Distance Formula:***

Distance between any two points (x_1, y_1) ***and*** (x_2, x_2)

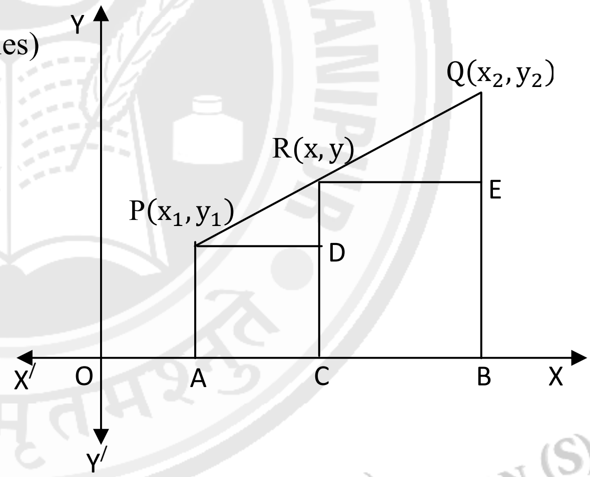
$$= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

❖ ***Section Formula:***

The coordinates of the point which divides the line joining the points (x_1, y_1) ***and*** (x_2, y_2) ***in the ratio m:n are*** $(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n})$.

Proof:- Let $X^{/}OX$ and $Y^{/}OY$ be the coordinate axes so that O is the origin. Let $R(x, y)$ divides the join of $P(x_1, y_1)$ and $Q(x_2, y_2)$ internally in the ratio m:n.

PA, QB and RC are drawn perpendicular to x-axis. PD⊥RC and RE⊥QB are also drawn.



In ΔPDR and ΔREQ, we have

∠PDR=∠REQ = 90^0

∠RPD = ∠QRE (being corresponding angles)

∴ ΔPDR~ΔREQ (by AA similarity)

Then, $\frac{PR}{RQ} = \frac{PD}{RE} = \frac{RD}{QE}$

$i.e.\ \frac{m}{n} = \frac{x-x_1}{x_2-x} = \frac{y-y_1}{y_2-y}$

Considering $\frac{m}{n} = \frac{x-x_1}{x_2-x}$, we have

$$n(x - x_1) = m(x_2 - x)$$

$$\Rightarrow nx - nx_1 = mx_2 - mx$$

$$\Rightarrow mx + nx = mx_2 + nx_1$$

$$\Rightarrow (m + n)x = mx_2 + nx_1$$

$$\Rightarrow x = \frac{mx_2+nx_1}{m+n}$$

In the same manner, considering $\frac{m}{n} = \frac{y-y_1}{y_2-y}$, we can write

$$y = \frac{my_2+ny_1}{m+n}$$

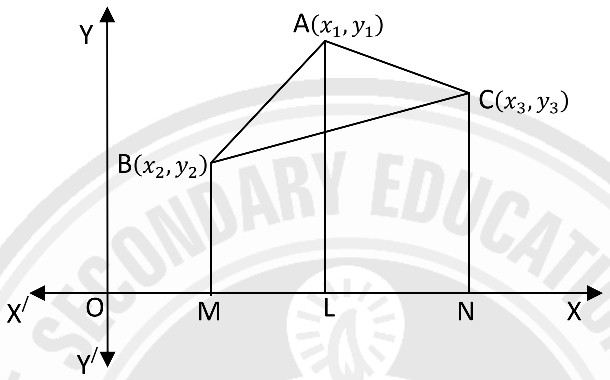
Thus, the coordinates of the point R which divides the join of $P(x_1, y_1)$ and $Q(x_2, y_2)$ internally in the ratio m:n are $(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n})$.

Deduction: The mid-point of a line segment is the point which divides the line segment in the ratio 1 : 1. Thus, the coordinates of the mid-point of the line segment joining $P(x_1, y_1)$ and $Q(x_2, y_2)$ are $(\frac{1.x_2+1.x_1}{1+1}, \frac{1.y_2+1.y_1}{1+1})$ i.e. $(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2})$.

❖ *Area of a Triangle*

Area of a Triangle whose vertices are $(x_1, y_1), (x_2, y_2)$ ***and*** (x_3, y_3) ***is***
$\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$.

Proof:



Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ be the vertices of a ΔABC.

AL, BM and CN are drawn perpendicular to x-axis.

We know, Area of a trapezium

$= \frac{1}{2} \times$ sum of the patrallel sides $\times$ perpendicular distance between the parallel sides.

Now, Area of ΔABC

$=$ Area of trapezium ABML+ Area of trapezium ALNC – Area of trapezium BMNC

$= \frac{1}{2}(AL + BM)ML + \frac{1}{2}(AL + CN)LN - \frac{1}{2}(BM + CN)MN$

$= \frac{1}{2}(y_1 + y_2)(x_1 - x_2) + \frac{1}{2}(y_1 + y_3)(x_3 - x_1) - \frac{1}{2}(y_2 + y_3)(x_3 - x_2)$

$= \frac{1}{2}[(y_1 + y_2)x_1 - (y_1 + y_2)x_2 + (y_1 + y_3)x_3 - (y_1 + y_3)x_1 - (y_2 + y_3)x_3 + (y_2 + y_3)x_2]$

$= \frac{1}{2}[x_1(y_1 + y_2 - y_1 - y_3) + x_2(y_2 + y_3 - y_1 - y_2) + x_3(y_1 + y_3 - y_2 - y_3)]$

$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$.

> ***Note:***
>
> ***(i) In case, the area calculated is negative, the negative sign will be ignored i.e. the absolute value of the area is taken only because negative area is meaningless.***
>
> ***(ii) If the area of a ΔABC is zero, then the three vertices A, B, C are collinear and conversely.***

****