CLASS – X
MATHEMATICS
CHAPTER – 4
PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

NOTES

- **The general form of a pair of linear equations in two variables**

The general form of a pair of linear equations in two variables x and y is

$$a_1x + b_1y + c_1 = 0$$

$$\text{and } a_2x + b_2y + c_2 = 0$$

where $a_1, b_1, c_1, a_2, b_2, c_2$ are all real numbers and $a_1{}^2 + b_1{}^2 \neq 0, a_2{}^2 + b_2{}^2 \neq 0$.

- **Solution of a Pair of Linear Equations in Two Variables**

The ordered pair (x_1, y_1) is said to be a solution of a pair of linear equations in two variables x and y, if $x = x_1, y = y_1$ satisfy both the linear equations.

- **Consistent pair:** A pair of linear equations in two variables having at least one solution is called a consistent pair.
- **Inconsistent pair:** A pair of linear equations in two variables having no solution is called an inconsistent pair.
- **Dependent pair:** A pair of linear equations is said to be a dependent pair if one equation is obtained from the other on multiplying by a constant.

Note: A dependent pair of linear equations has infinitely many solutions. So, a dependent pair of equations is also a consistent pair.

- **Graphical Representation of a Pair of Linear equations**
  1. If the graphs (lines) of a pair of linear equations intersect at one point, there is a unique solution.
  2. If the lines of a pair of linear equations are coincident, there are infinitely many solutions i.e. the pair equations is dependent.
  3. If the lines of a pair of linear equations are parallel, there is no solution i.e. the pair of linear equations is inconsistent.

Note: The graphs of a consistent pair are either intersecting lines or coincident lines.

Comparison of the ratios of the coefficients of a pair of linear equations

In the pair of linear equations

$$a_1x + b_1y + c_1 = 0$$

and $a_2x + b_2y + c_2 = 0$

1. if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, the graphs (lines) are intersecting i.e. there is a unique solution.
2. if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, the lines are coincident i.e. the pair of equations has infinitely many solutions i.e. the pair of equations is dependent.
3. if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, the lines are parallel i.e. the pair of equations has no solution i.e. the pair of equations is inconsistent.

Note: Condition for consistent pair is either $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ or $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

Algebraic Method of Solving a Pair of Linear Equations

• Substitution Method

Steps to find the solution of a pair of equations in two variables by substitution method:

1. From either equation, whichever is convenient, find the value of one variable, say y in terms of the other variable i.e. x.
2. Substitute the value of y thus obtained in Step 1 in the other equation and reduce it to an equation in only one variable x, which can be solved; and hence obtain the value of x.

   Sometimes after substitution you may get an equality relation with no variable. If this relation is true, you can conclude that the pair of linear equations has infinitely many solutions. If the relation is false, then the pair of linear equations is inconsistent.

3. Substitute the value of x obtained in Step 2 in the equations of Step 1 and obtain the value of y.

• Elimination method

Steps to solve of a pair of equations in two variables by elimination method:

1. Multiply or divide both the equations by suitable non-zero constant so that the coefficients of one variable (either x or y) become numerically equal.
2. Then add one equation to the other or subtract one from the other so that one variable gets eliminated. If we get an equation in one variable, go to Step 3.

   If we obtain a true equality relation involving no variable, then the original pair of equations has infinitely many solutions.

If we obtain a false relation involving no variable, then the original pair of equations has no solution.

3. Solve the equation obtained in Step 2 and obtain the value of the variable which is not eliminated.
4. Substitute the value of the variable obtained in Step 3 in any of the given equations to get the value of the other variable.

- **Cross-Multiplication Method**

The pair of linear equations

$$a_1x + b_1y + c_1 = 0$$

and $a_2x + b_2y + c_2 = 0$ can be combined as

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1},\ a_1b_2 - a_2b_1 \neq 0.$$

Steps to find the solution of a pair of linear equations in two variables by Cross-Multiplication Method:

1. Write the given equations in the general form.
2. Write the pair of equations as $\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$.
3. From the equations obtained in Step 2, find the values of x and y, provided $a_1b_2 - a_2b_1 \neq 0$

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