CHAPTER 1
SETS, RELATIONS AND FUNCTIONS

NOTES

Set:

A well-defined collection of distinct objects is called a set.

Representation Of Sets

1. **Roaster or Tabular form:** In this method all the elements are listed, the elements are being separated by commas and are enclosed within braces { }. For example, the set of all even positive integers less than 7 is described in roaster form as $\{2,\ 4,\ 6\}$.
2. **Set builder form:** In this method all the elements of a set possess a single common property and this property is represented by a single variable. For example in the set $\{a, e, i, o, u\}$ all the elements possess a common property. Namely, each of them is a vowel in English alphabets. Denoting this set by V, we write V = $\{x : x$ is a vowel in English Alphabet$\}$.

Empty set : A set having no element is called a null set or empty set or void set .

It is denoted by ϕ or { }

Finite set : A set which is empty or consist of elements that can be count is called finite set.

Infinite set : A set consist of elements that cannot be count is called an infinite set.

Singleton set : A set having exactly one element is called singleton set.

Non empty set : A set having at least one element is called non empty set.

Subsets : A set X is said to be a subset of set Y if every element of X is also an element of Y. It is express as $X \subseteq Y$. Y is said to be a superset of set X.

Equality of sets : Two sets A and B are said to be equal if they have exactly the same elements and we write as A=B. Otherwise the sets are said to be unequal and we write $A \neq B$

Proper subsets : B is said to be a proper subset of set A if and only if all elements in B are in A and there exists at least one element in A which is not in B. We write $B \subset A$ to mean B is a proper subset of A.

Improper subsets : A subset of a set is said to be improper if it is not proper.

Power set : The collection of all subsets of a set A is called the power set of A. It is denoted by P(A).

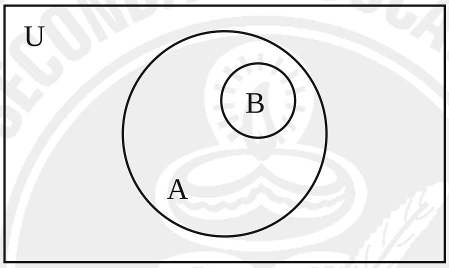
Set of sets : A set whose elements are sets themselves, is known as set of sets.

Equivalent sets : Two finite sets A and B are said to be equivalent if n(A) = n(B)

where n(A) = numbers of elements in A, n(B)= numbers of elements in B.

Universal Sets : A set that contains all the sets under consideration is called Universal Set. It is denoted by U or S or ξ .

Venn Diagram : Pictorial representation of sets and its operation is known as Venn Diagram. In this diagram, a universal set is represented by the points in a rectangle and its subsets by the points within circles drawn inside the rectangle. e.g.



$B \subseteq A$

Union of sets : Union of two sets A and B is a set whose elements are either in set A or in set B or in both the sets A and B. It is denoted by $A \cup B$.

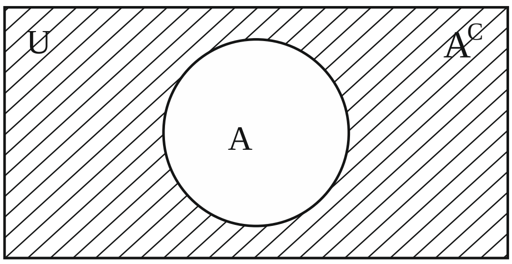
Intersection of sets: Intersection of two sets A and B is the set X whose elements are common to both the sets. In symbols, we write $X = A \cap B$

Disjoint sets : Two sets A and B are said to be disjoint or non-overlapping if they have no elements in common.

i.e. $A \cap B = \phi$, then A and B are disjoint sets.

Difference of sets : The difference of two sets A and B is a new set which contains all those elements which belongs to 'A' and not to 'B'. It is denoted by A – B and B – A = set of element of B which are not in A.

Complements of sets: The complement of set A is the set of all elements of universal set which are not the elements of A. Complement of set A is written as $A^{/}$ or A^{C}.



De Morgan' Laws of complementation:

i) $(A \cup B)' = A' \cap B'$ ii) $(A \cap B)' = A' \cup B'$

Some basic facts about complementation:

a) $(A')' = A$ b) (i) $U' = \phi$ and ii) $\emptyset' = U$

c) (i) $A \cap A' = \phi$ and (ii) $A \cup A' = U$

d) $A \subseteq B \Leftrightarrow B' \subseteq A'$

Cardinal Numbers of finite sets: If A be a finite set then the numbers of elements in A, denoted by n(A) is called cardinal number of cardinality of A.

Cartesian product of two sets: Given two non-empty sets A and B. The Cartesian product AXB is the set of all ordered pairs of elements as (x, y) where x from A and as first component and y from B as second component, which can be written in set-builder notation as

$$A \times B = \{(x, y): x \in A \text{ and } y \in B\}$$

Relation : Let A and B be the two given non-empty sets. A relation from A to B is the subset of AXB.

If $A = \{1, 2, 3\}$, $B = \{4, 5\}$

$$A \times B = \{(1,4), (1,5), (2,4), (2,5), (3,4), (3,5)\}$$

Then $R = \{(1,4), (2,5), (3,4)\}$ is a relation from A to B.

Domain of a relation: The set of all first elements of order pairs in a relation R from a set A to set B is called the domain of the relation R.

Range of the relation: The set of all second elements of order pairs in a relation R from a set A to B is called the range of the relation R.

Inverse Relations: Let R be a relation from a set A to a set B. The relation R^{-1} from B to A given by $R^{-1} = \{(y, x) : (x, y) \in R\}$ is said to be inverse relation from B to A.

Relation in a set: Let A be a non-empty set. Then a relation from A to A i.e. a subset of A×A is called a relation in a set A.

Types of relations: **(i) Reflexive:** A relation R is said to be reflexive if $(a, a) \in R$ for all $a \in A$ i.e. if each element of A is related to itself.

(ii) Symmetric relation: A relation R is said to be a symmetric if whenever *a* is related to *b*, *b* is related to *a* i.e. $(a, b) \in R \Rightarrow (b, a) \in R$ for any $a, b \in A$.

(iii) Transitive relation: A relation R is said to be transition if a is related to b and b is related to $c \Rightarrow a$ is related to c. *i.e. if* $(a,b) \in R$ and $(b,c) \in R \Rightarrow (a,c) \in R$

Equivalence Relation: A relation R in a set A is said to be an equivalence relation if R is reflexive, symmetric and transitive.

Identity Relation: A relation R in a set A is said to be identity if each element of A is related to itself only. i.e. $R = \{(a,a) : a \in A\}$.

Universal Relation: For any set A, the product $A \times A$ is a relation in A. This relation is called the universal relation in A.

Function or Mapping: Let A and B be any two non-empty sets then a function *f* from A to B is defined as a relation from A to B satisfying the following conditions

(i) $\forall a \in A$ there exist $b \in B$ such that $(a,b) \in f$

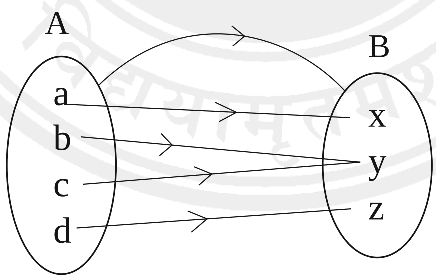
(ii) $(a,b) \in f$ and $(a,c) \in f \Rightarrow b = c$

Domain of a function: If *f* is a function from A to B then the set A is called domain and B is called the co-domain of the function *f*.

Range of a function: If *f* is a function from A to B, then the set of all *f*-images under the function *f* is called range of the function *f*.

Diagramatic Representation of a function:

A mapping $f : A \to B$ may be represented by diagram



Types of mapping or functions:

Injective or one-one mapping:

A mapping $f : A \to B$ is said to be one-one or injective if different elements of A have the different *f*-images in *B*.

Many one mapping: A mapping $f : A \to B$ is said to be many-one if there exists atleast two distinct elements in A having the same *f*-images in B.

Onto or surjective mapping: A mapping $f : A \to B$ is said to be onto or surjective if each element in B is an *f*-image of at least one element in A.

Into mapping: A mapping $f : A \rightarrow B$ is said to be into if there exist at least one element in B which is not f-image of any element of A.

One-one onto or bijective mapping:

A mapping $f : A \rightarrow B$ is said to be one-one onto or bijective if f is one-one as well as onto.

Constant functions: A mapping $f : A \rightarrow B$ is called a constant function if its range consists of exactly one element of B.

i.e. if f(A) ={y}, for some $y \in B$.

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