CHAPTER 3
MATHEMATICAL LOGIC

NOTES

MATHEMATICAL LOGIC : In Mathematics, the proofs are based on sound reasoning or arguments. The main object of logic is to provide a set of rules by which one can determine whether any particular reasoning or argument is valid or not.

STATEMENT : A statement is an assertive sentence which is either true or false but not both simultaneously.

E.g. : **(i)** 5 is a prime number. (True)

(ii) The square of every real number is negative. (False)

TRUTH VALUE : The truth or falsity of a statement is called its truth value. The truth value of a true statement is denoted by T and the truth value of false statement is denoted F.

SIMPLE STATEMENT : A statement which does not contain any other statement as its part is called a simple or atomic or a primary statement.

E.g. : A triangle has three sides.

COMPOUND STATEMENT : A statement formed by combining two or more simple statements is called a compound or molecular statement.

E.g. : Roses are red and sky is blue.

LOGICAL CONNECTIVES : The words or phrases which combine simple statements to form compound statements are known as sentential or logical connectives.

Main logical connectives are ‘And’, ‘Or’, ‘Not’, ‘If…… then’, and ‘If and only if’.

CONJUNCTION : The compound statement formed by combining two statements using the connective ‘and’ is called conjunction of the two statements. Thus, if *p* and *q* are two statements then their conjunction is ‘*p* and *q*’ and is denoted by the symbol ‘ $p \wedge q$ ’. Truth table for $p \wedge q$:

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

DISJUNCTION : The compound statement formed by combining the two statements using the connective 'or', is called disjunction of the two statements. The disjunction of the statements p and q is denoted by '$p \vee q$' and read as 'p or q'. Truth table for $p \vee q$.

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

NEGATION : Negation is the denial of the assertion made in the statement. The negation of a statement p is denoted by $\sim p$. The truth table is given below:

p	$\sim p$
T	F
F	T

IMPLICATION OR CONDITIONAL

: The compound statement formed by conjoining two statements by the connective 'If…….then', is called an implication or a conditional.

E.g.:- 'If it rains, then there will be no play'.

If p and q are the statements, then the implication 'If p then q' is denoted by "$p \Rightarrow q$" (read as 'p implies q'). The truth table for $p \Rightarrow q$ is

p	q	$p \Rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

DOUBLE IMPLICATION OR BICONDITIONAL STATEMENT

: The compound statement formed by conjoining two statements by the connective 'If and only if' is called a double implication or a bi-conditional statement.

If p and q are any two statements, then 'p if and only if q' or 'p iff q' is denoted by '$p \Leftrightarrow q$'. The truth table for '$p \Leftrightarrow q$' is

p	Q	$p \Leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

OPEN STATEMENT

: An open statement is an assertive sentence involving one or more unknowns or unspecified terms and whose truth value depends or these unknowns or unspecified terms.

E.g.:- He is an ex-principal of the college.

Negations of compound statements:

a) The negation of $p \wedge q$ is $\sim p \vee \sim q$.

b) The negation of $p \vee q$ is $\sim p \wedge \sim q$.

c) The negation of $\sim p$ is p.

d) The negation of $p \Rightarrow q$ is $p \wedge \sim q$.

e) The negation of $p \Leftrightarrow q$ is $\sim p \Leftrightarrow q$ or $p \Leftrightarrow \sim q$.

CONVERSE OF AN IMPLICATION

: The converse of a given implication is a new implication formed by interchanging the antecedent and the consequent of the given implication. Thus, the converse of $p \Rightarrow q$ is $q \Rightarrow p$.

TAUTOLOGY

: A compound statement which is always true irrespective of the truth values of its component parts is called a tautology.

CONTRADICTION

: A compound statement which is always false irrespective of the truth values of its component parts is called a contradiction.

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