# CHAPTER 7
# VECTORS

## NOTES

The physical quantities may be divided into two groups:

**(i)** Scalars **(ii)** Vectors

**(i) Scalar :** A scalar is a quantity having magnitude but no direction. For example: mass, length, time, temperature, volume, density, work etc.

**(ii) Vectors :** A vector is a quantity having both magnitude as well as direction. For example: force, velocity, acceleration, displacement, moment etc.

## Representation of a vector:

A directed line segment from A to B represents a vector and is written as $\overrightarrow{AB}$. A is the initial point and B is the terminal point of the vector $\overrightarrow{AB}$. Symbols like $\vec{a}, \vec{b}, \vec{c}$........ etc. with arrow overhead are used to denote vectors.

## TYPES OF VECTORS:

**Zero (or null) Vector :** A vector whose magnitude is 0 (zero) is called a zero vector. It can have any arbitrary direction. For a zero vector the initial and terminal points are coincident.

**Proper vector :** Any vector other than a zero vector is called a proper vector.

**Co-initial vectors :** Vectors are said to be co-initial if they have the same initial point.

**Co-terminus vectors :** Vectors are said to be co-terminus if they have the same terminal point.

**Equal vectors :** Two vectors $\vec{a}$ and $\vec{b}$ are said to be equal if they have equal magnitude and are equally directed and it is written as $\vec{a} = \vec{b}$.

**Like vectors :** Vectors are said to be like if they are equally directed irrespective of their magnitude.

**Unlike vectors :** Vectors are said to be unlike when they have opposite directions irrespective of their magnitude.

**Negative of a vectors :** The vector, which has the same magnitude as the vector $\vec{a}$ but opposite direction, is called the negative of $\vec{a}$ and is denoted by $-\vec{a}$.

$$\text{Thus if } \overrightarrow{AB} = \vec{a}, \text{ then } \overrightarrow{BA} = -\vec{a}.$$

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**Scalar multiplication of a vector:** Let $n$ be a scalar and $\vec{a}$ be any vector. The product $n\vec{a}$ is a vector, whose magnitude is $|n|$ times that of $|\vec{a}|$ and the direction is same or opposite direction as that of $\vec{a}$ according as $n > 0$ *or* $n < 0$.

**Unit vector** : A vector whose magnitude is one is called a unit vector. For any proper vector $\vec{a}$, $\frac{\vec{a}}{|\vec{a}|}$ is a unit vector in the direction of $\vec{a}$.

**Note:** Usually we use the symbol $\hat{a}$ (a cap) to denote a unit vector.

$$\text{Thus } \hat{a} = \frac{\vec{a}}{|\vec{a}|}$$

$$\Rightarrow |\vec{a}|\,\hat{a} = \vec{a}$$

$$\Rightarrow a.\hat{a} = \vec{a}$$

**Reciprocal of a vector** : For any proper vector $\vec{a}$, the vector having the same direction as that of $\vec{a}$ but whose magnitude is the reciprocal of the magnitude of $\vec{a}$ is called the reciprocal vector of $\vec{a}$.

**Note** : The Vector $\frac{\vec{a}}{a^2}$ is the reciprocal of $\vec{a}$.

**Free vectors** : A vectors whose initial point is not specified but whose magnitude and direction are known is called free vector.

**Localized vectors** : A vector drawn parallel to a given vector, but through a specified point as initial point, is called localized vector.

**Position vectors** : The vector identifying the position of a point $P$ in space with reference to a particular point is called the position vector of the point $P$. In general, we take the origin $O$ in the system of Cartesian co-ordinates as the point of reference. Thus the vector $\vec{r}$ is given by $\vec{r} = \overrightarrow{OP}$ is the position vector of $P$.

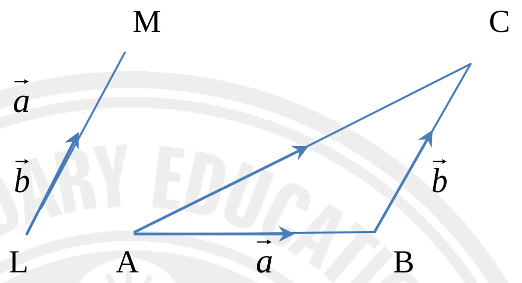
## Addition of two vectors:

Let $\vec{a} = \overrightarrow{AB}$ and $\vec{b} = \overrightarrow{LM}$ be two vectors as shown in the fig. To find the sum of $\vec{a}$ and $\vec{b}$, we draw $\overrightarrow{BC} = \overrightarrow{LM} = \vec{b}$ such that the initial point of $\overrightarrow{BC}$ is the terminal point of $\overrightarrow{AB}$.

If $\overrightarrow{AC} = \vec{c}$ then it is defined as the sum of $\vec{a}$

and $\vec{b}$ and we write $\vec{a} + \vec{b} = \vec{c}$

Or $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$



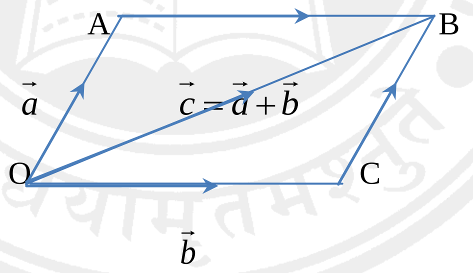

This is also referred to as the triangle law of vectors addition.

## Parallelogram law of vectors addition:

The sum of vectors $\vec{a}$ and $\vec{b}$ can also be obtained by bringing the initial points of $\vec{a}$ and $\vec{b}$ together and then completing the parallelogram OABC, the diagonal $\overrightarrow{OB}$ which is co-initial with $\vec{a}$ and $\vec{b}$ is sum of $\vec{a}$ and $\vec{b}$.

i.e. $\vec{a} + \vec{b} = \vec{c}$

$\overrightarrow{OA} + \overrightarrow{OC} = \overrightarrow{OB}$




## Deductions:

**1.** Vector addition is commutative: For any two vectors $\vec{a}$ and $\vec{b}$

$$\vec{a} + \vec{b} = \vec{b} + \vec{a}$$

**2.** Vector addition is associative: For any three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$

$$\left(\vec{a} + \vec{b}\right) + \vec{c} = \vec{a} + \left(\vec{b} + \vec{c}\right)$$

**3.** For any two vectors $\vec{a}$ and $\vec{b}$, $|\vec{a} + \vec{b}| \leq |\vec{a}| + |\vec{b}|$.

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**Subtraction of vectors:**

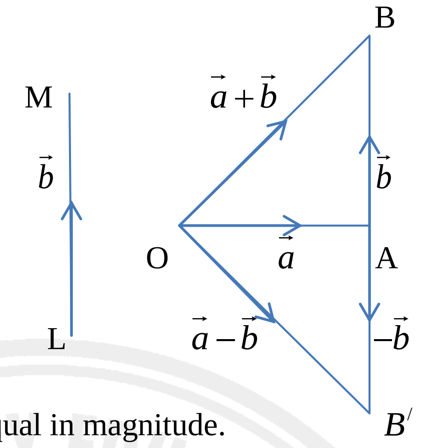
In fig. $\overrightarrow{OA}$ is the vector $\vec{a}$ and $\overrightarrow{LM} = \vec{b}$.

We draw $\overrightarrow{AB} = \overrightarrow{LM}$ so that $\overrightarrow{AB} = \vec{b}$.

Now $\overrightarrow{OB} = \vec{a} + \vec{b}$

To find $\vec{a} + (-\vec{b})$ i.e. $\vec{a} - \vec{b}$ we draw

$\overrightarrow{AB'} = -\overrightarrow{LM}$ reversing the direction of $\vec{b}$ but equal in magnitude.




So that $\overrightarrow{AB'} = -\vec{b}$

Then $\overrightarrow{OB'} = \vec{a} - \vec{b}$

**Notes:** **1)** If $\vec{a}$ and $\vec{b}$ are perpendicular, then $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$.

**2)** Like algebraic quantities vectors can be transported across the sign of equality.

i.e. if $\vec{a} + \vec{b} = \vec{c}$, then $\vec{a} = \vec{c} - \vec{b}$.

**3)** Distributive law of scalar multiplication: $m(\vec{a} + \vec{b}) = m\vec{a} + m\vec{b}$.

- **Representation of a vector in terms of position vectors of its end points:**

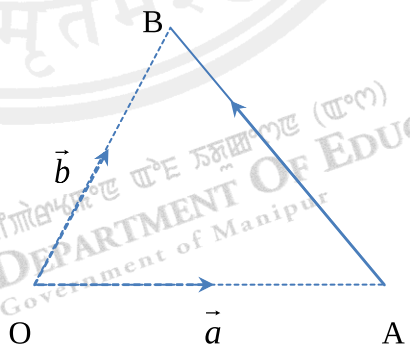
**Let $\vec{a}$ and $\vec{b}$ be the position vectors of points $A$ and $B$ respectively.** Then, $\overrightarrow{OA} = \vec{a}$,

$\overrightarrow{OB} = \vec{b}$

In $\Delta OAB$, we have

$\overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OB}$

$\Rightarrow \overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \vec{b} - \vec{a}$




$\Rightarrow \overrightarrow{AB} =$ (Position vector of B)

$\Rightarrow \overrightarrow{AB} =$ (Position vector of head) – (Position vector of tail)

## SECTION FORMULA:

**Theorem 1 (Internal Division):** Let A and B be the two points with position vectors $\vec{a}$ and $\vec{b}$ respectively, and let *C* be a point dividing *AB* internally in the ratio *m:n*. Then the position vector of *C* is given by $\dfrac{m\vec{b}+n\vec{a}}{m+n}$

**Proof:** Let *O* be the origin of reference, then $\overrightarrow{OA}=\vec{a}$ and $\overrightarrow{OB}=\vec{b}$.

Let $\vec{c}$ be the position vector of *C* which divides *AB* internally in the ratio of *m:n*. Then

$$\frac{AC}{CB}=\frac{m}{n}$$

$$\Rightarrow n.AC=m.CB$$

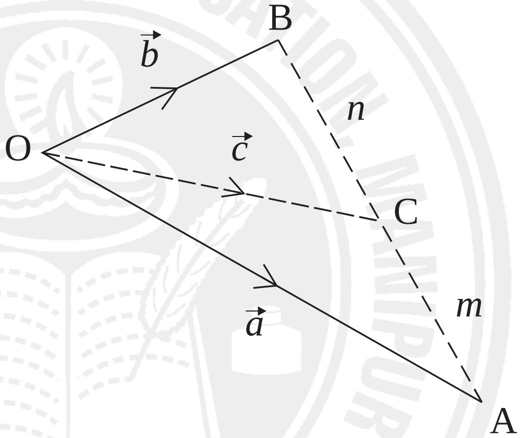
$$\Rightarrow n.\overrightarrow{AC}=m.\overrightarrow{CB}$$

$$\Rightarrow n(\vec{c}-\vec{a})=m(\vec{b}-\vec{c})$$

$$\Rightarrow n\vec{c}+m\vec{c}=m\vec{b}+n\vec{a}$$

$$\Rightarrow (m+n)\vec{c}=m\vec{b}+n\vec{a}$$

$$\Rightarrow \vec{c}=\frac{m\vec{b}+n\vec{a}}{m+n}\Rightarrow \overrightarrow{OC}=\frac{m\vec{b}+n\vec{a}}{m+n}$$




Hence the position vector of point *C* is $\dfrac{m\vec{b}+n\vec{a}}{m+n}$

### Deduction:

**If *C* is the mid-point of AB, then it divides *AB* in the ratio 1:1. Therefore position vector of *C* given by**

$$\frac{1.\vec{a}+1.\vec{b}}{1+1}=\frac{\vec{a}+\vec{b}}{2}=\frac{1}{2}\left(\vec{a}+\vec{b}\right)$$

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