CHAPTER- 1
NUMBER SYSTEM

NOTES

Natural Numbers: 1, 2, 3, ……………………………………

Whole Numbers: 0, 1, 2, 3, …………………………………

Integers: $\ldots\ldots\ldots\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\ldots\ldots\ldots$

- **Rational number**

  It is a number which can be expressed in the form of $\frac{p}{q}$ where p, q are integers and $q \neq 0$.

- Every natural number is a whole number, every whole number is an integer and every integer is a rational number.
- If a and b are two rational number such that $a<b$, then $\frac{a+b}{2}$ is a rational number lying between a and b.
- If $\frac{p}{q}$ and $\frac{r}{s}$ are any two rational numbers and $\frac{p}{q} < \frac{r}{s}$ then $\frac{p}{q} < \frac{p+r}{q+s} < \frac{r}{s}$.
- There are an infinite number of rational numbers between two unequal rational numbers.
- **The nth root of a Real number**

  If x is a positive real number and n is a positive integer then $\sqrt[n]{x} = y$ means $y^n = x$ and $y>0$. $\sqrt[n]{x}$ is called nth root of the positive number x.

  - **Prove that $\sqrt{2}$ is not a rational number.**

  Proof :- Let us suppose that $\sqrt{2}$ is a rational number. Then there exists integers p and q such that $q \neq 0$, p , q are co-prime and $\frac{p}{q} = \sqrt{2}$

  $\therefore (\frac{p}{q})^2 = (\sqrt{2})^2$ [Squaring both sides]

  $\Rightarrow \frac{p^2}{q^2} = 2$

  $\Rightarrow \frac{p^2}{q.q} = 2$

  $\Rightarrow \frac{p^2}{q} = 2q$ ……………………..(1)

As p and q are co-prime, the left side of (1) is a fraction except for $q = 1$ but the right side is an integer.

If $q = 1, \frac{p^2}{1}=2\times1$

$\Rightarrow p^2=2$ in which both sides are integers.

It means that the square of p is 2. But there is no integer whose square is 2.

$[\because 1^2 < 2 < 2^2]$

So, our supposition that $\sqrt{2}$ is a rational number is contradicted.

Hence $\sqrt{2}$ is not a rational number.

Laws of exponents

If m, n are integers and x, y are non-zero rational numbers, then

i) $x^m \times x^n = x^{m+n}$

ii) $\frac{x^m}{x^n} = x^{m-n}$

iii) $(xy)^m = x^m y^m$

iv) $(x^m)^n = x^{mn}$

The above Laws of exponents hold good when bases are positive real numbers and exponents are rational numbers, positive or negative.

Show that $x^0=1$ for any non zero rational number x.

Solution:- We know that

$$\frac{x^m}{x^n} = x^{m-n}$$

If $m = n$,

$$\frac{x^n}{x^n} = x^{n-n}$$

$$\Rightarrow 1 = x^0$$

$$\therefore x^0 = 1$$

Show that $x^{-n}=\frac{1}{x^n}$ for any non –zero rational number x.

Solution :- We know that,

$$\frac{x^m}{x^n} = x^{m-n}$$

If $m = 0$,

$$\frac{x^0}{x^n} = x^{o-n}$$

$$\Rightarrow \frac{1}{x^n} = x^{-n}$$

$$\therefore x^{-n} = \frac{1}{x^n}$$

- **Dedekind-Cantor Axiom**

  "To every real number there corresponds a unique point on the number line and to every point on the number line there corresponds a unique real number."

- **Irrational numbers**

  Irrational numbers are numbers represented on the number line by points other than those representing rational numbers.

- **Real numbers:** Real numbers are numbers which are either rational or irrational.

- **Rationalising factor**

  If the product of two irrational numbers is a rational number then each is called a rationalising factor of the other.

Example:- $2 + \sqrt{2}$ and $2 - \sqrt{2}$ are rationalising factor of each other.

For $(2 + \sqrt{2})(2 - \sqrt{2}) = 2^2 - (\sqrt{2})^2 = 4 - 2 = 2$, which is a rational number.

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